A Simple Error Correction Method for Two-Port Transmission Parameter Measurement

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***Abstract*—This letter presents a simple error correction method for two-port transmission parameter measurement. The method requires measuring only two nonreflecting transmission lines instead of a complete set of calibration standards (at least three) for error correction. It does not need an explicit solution of error coefficients. These two features make it simpler than any other methods. Error-corrected measurement results of a CPW discontinuity using the new method and the TRL method are in good agreement. The new method is useful in characterizing well-matched two-port devices or those whose reflection parameters are of little interest.**

***Index Terms*— Calibration, scattering parameters measurement.**

I. Introduction

It IS generally believed that to obtain error-corrected measurement of one or all the scattering parameters of a two-port device, one needs to perform a full two-port calibration using a set of standards before measuring the device. So, much effort has been made to generate accurate calibration techniques [1]–[3].

However, if the device can be characterized electrically by one parameter like propagation constant or permittivity, error-corrected measurement of such a parameter can be extracted from two raw S-matrix measurements, without requiring a full two-port calibration. One example is a section of nonreflecting transmission line [4]. Another example is a length of coaxial line or waveguide uniformly filled with a nonmagnetic material [5]–[7]. An interesting question is: Can we still extract error-corrected transmission parameters of a two-port device without making a complete two-port calibration? The answer is "Yes." The reason and supporting results are presented in Sections II and III, respectively. Conclusions and discussions are given in the last section.

II. Theory

The method presented here produces error-corrected transmission parameters of a two-port device using two nonreflecting transmission lines along with the device. The principle of the error correction is based on an eight-term error model of a two-port measurement system which uses two error-two-ports to represent removable errors. The wave cascading matrix (WCM) description [1] is very useful in calibration/error removing problems. The WCM of a two-port device is defined as

$$\boldsymbol{R} = \frac{1}{s_{21}}\begin{pmatrix} -\Delta & s_{11} \\ -s_{22} & 1 \end{pmatrix} \tag{1}$$

$$\Delta = s_{11}s_{22} - s_{12}s_{21}. \tag{2}$$

In (1) and (2), s_{mn} is the s-parameters of the device.

With the error model and the WCM description, the measurement of a zero-length nonreflecting line embedded between error-two-port x and y corresponds to

$$\boldsymbol{R}_t = \boldsymbol{R}_x\boldsymbol{R}_y. \tag{3}$$

Similarly, the measurement of another nonreflecting line of length l gives

$$\boldsymbol{R}_l = \boldsymbol{R}_x\begin{pmatrix} e^{-\gamma l} & 0 \\ 0 & e^{\gamma l} \end{pmatrix}\boldsymbol{R}_y \tag{4}$$

where γ is the complex propagation constant of a wave in the line while the measurement of the device yields

$$\boldsymbol{R}_d = \boldsymbol{R}_x\frac{1}{s_{21}}\begin{pmatrix} -\Delta & s_{11} \\ -s_{22} & 1 \end{pmatrix}\boldsymbol{R}_y \tag{5}$$

where Δ is expressed in (2).

To remove the influence of the two error-two-ports on the device parameters, a simple procedure based on the calculation of the determinant of a matrix product is employed. The mathematical basis for the calculation is that the determinant of a product is equal to the product of the determinants. For the purpose of using that basis, two of the three matrix equations (3)–(5) are taken and added, producing three new equations as follows:

$$\boldsymbol{R}_t + \boldsymbol{R}_l = \boldsymbol{R}_x\begin{pmatrix} 1+e^{-\gamma l} & 0 \\ 0 & 1+e^{\gamma l} \end{pmatrix}\boldsymbol{R}_y \tag{6}$$

$$\boldsymbol{R}_t + \boldsymbol{R}_d = \boldsymbol{R}_x\begin{pmatrix} 1-\dfrac{\Delta}{s_{21}} & \dfrac{s_{11}}{s_{21}} \\ -\dfrac{s_{22}}{s_{21}} & 1+\dfrac{1}{s_{21}} \end{pmatrix}\boldsymbol{R}_y \tag{7}$$

$$\boldsymbol{R}_l + \boldsymbol{R}_d = \boldsymbol{R}_x\begin{pmatrix} e^{-\gamma l}-\dfrac{\Delta}{s_{21}} & \dfrac{s_{11}}{s_{21}} \\ -\dfrac{s_{22}}{s_{21}} & e^{\gamma l}+\dfrac{1}{s_{21}} \end{pmatrix}\boldsymbol{R}_y. \tag{8}$$

Then, taking the determinant of the resulting matrices on both sides of these new equations and dividing them by the determinant of $\boldsymbol{R}_t$ (or $\boldsymbol{R}_l$ or $\boldsymbol{R}_d$) generates

$$(1+e^{-\gamma l})(1+e^{\gamma l}) = \frac{\det(\boldsymbol{R}_t + \boldsymbol{R}_l)}{\det(\boldsymbol{R}_t)} \tag{9}$$

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$$\left(1-\frac{\Delta}{s_{21}}\right)\left(1+\frac{1}{s_{21}}\right)+\frac{s_{11}s_{22}}{s_{21}^2}=\frac{\det(\boldsymbol{R}_t+\boldsymbol{R}_d)}{\det(\boldsymbol{R}_t)} \tag{10}$$

$$\left(e^{-\gamma l}-\frac{\Delta}{s_{21}}\right)\left(e^{\gamma l}+\frac{1}{s_{21}}\right)+\frac{s_{11}s_{22}}{s_{21}^2}=\frac{\det(\boldsymbol{R}_l+\boldsymbol{R}_d)}{\det(\boldsymbol{R}_t)}. \tag{11}$$

In (9)–(11), det means "taking the determinant of." Solving (9) for γ gives

$$\gamma=\frac{1}{l}\cosh^{-1}\left[\frac{\det(\boldsymbol{R}_t+\boldsymbol{R}_l)}{2\det(\boldsymbol{R}_t)}-1\right]. \tag{12}$$

Whether the device is reciprocal or not, one can always obtain

$$\frac{s_{12}}{s_{21}}=\frac{\det(\boldsymbol{R}_d)}{\det(\boldsymbol{R}_t)}. \tag{13}$$

Substituting (13) into (10) and (11) and solving them for s_{21} and Δ produces (14), shown at the bottom of the page, and

$$\Delta=1-\left[\frac{\det(\boldsymbol{R}_t+\boldsymbol{R}_d)-\det(\boldsymbol{R}_d)}{\det(\boldsymbol{R}_t)}-1\right]s_{21} \tag{15}$$

s_{12} can be easily calculated by (13) upon obtaining s_{21}. If the device is reflection-symmetrical $(s_{11}=s_{22})$, we have

$$s_{11}=\pm\sqrt{\Delta+s_{12}s_{21}} \tag{16}$$

where the sign ambiguity is to be resolved using a prior estimate.

III. Measurement Results

In order to validate the proposed method, a CPW discontinuity and a set of TRL calibration standards patterned on an indium phosphide substrate were measured using a microwave wafer probe station along with a vector network analyzer in the frequency range of 10–50 GHz. The measured raw S-matrices were processed to give the insertion loss and phase of the discontinuity using the new method and the TRL method [1]. The results are shown in Fig. 1 for comparison. It is found that the error-corrected data based on both methods agree very well with each other over the whole frequency band. The small discrepancy is due to the fact that the new method uses less measurements and less computations than the TRL method.

IV. Conclusions and Discussions

A simple method for error correction in two-port transmission parameter measurement has been presented. The method removes systematic errors using only two transmission line measurements instead of more measurements usually required by any other methods. Its validity has been illustrated by experimental results and its accuracy has been found to be comparable with that of the TRL method. Because of its simplicity and high accuracy, the method is useful in measuring the transmission parameters of two-port devices.

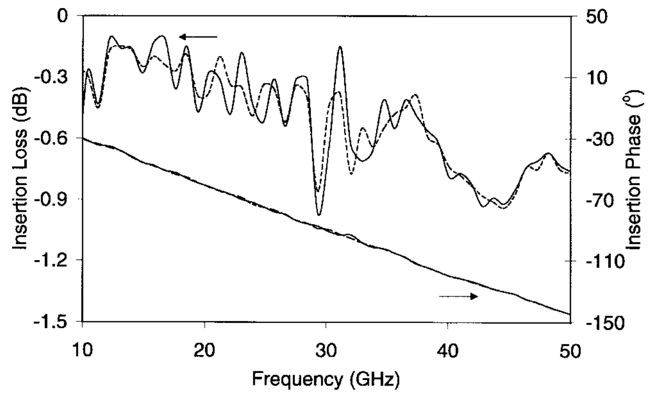

Fig. 1. Error-corrected insertion loss and phase using the new method (solid line) and the TRL method (dashed line).

It is interesting to note that the method of using three S-matrix measurement equations to determine three unknowns, presented here, is applicable to the simultaneous determination of complex permittivity and permeability of general materials. The results about this problem will be reported in another paper.

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$$s_{21}=\frac{2\sinh(\gamma l)}{\left[\dfrac{\det(\boldsymbol{R}_t+\boldsymbol{R}_d)-\det(\boldsymbol{R}_d)}{\det(\boldsymbol{R}_t)}-1\right]e^{\gamma l}-\dfrac{\det(\boldsymbol{R}_l+\boldsymbol{R}_d)-\det(\boldsymbol{R}_d)}{\det(\boldsymbol{R}_t)}+1} \tag{14}$$